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ON ABSTRACT VOLTERRA EQUATIONS WITH KERNELS HAVING A POSITIVE RESOLVENT

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ON ABSTRACT VOLTERRA EQUATIONS WITH KERNELS HAVING A POSITIVE RESOLVENT

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ABSTRACT

~~We consider~~ the nonlinear abstract Volterra equation of convolution type is considered.

$$\text{(V)} \qquad u(t) + b * Au(t) \ni u_0 + b * g(t) \qquad t \geq 0$$

where A is ~~m~~-accretive in a Banach space X, b is a given real kernel, $u_0 \in \overline{D(A)}$ and $g: [0,\infty) \to X$ are given.

Boundedness and asymptotic properties of the solutions are established under the assumption that the kernel satisfies certain natural positivity conditions. → to p. -B-

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Significance and Explanation

cont An important property of linear and nonlinear diffusion equations is that the solutions of such equations obey a "maximum principle". In particular, if the initial data and the forcing terms are nonnegative, then the solution is nonnegative.

Equation (V) stated in the Abstract is a generalization of the evolution equation

$$\text{(DE)} \quad \begin{cases} \dfrac{du}{dt} + Au = g & t > 0 \\ u(0) = u_0 . \end{cases}$$

For example, if $Au(x) = -\Delta u(x)$ with Dirichlet or Neumann conditions on a bounded domain of $\mathbb{R}^n$, then equation (DE) is the standard heat equation and the solution u is nonnegative whenever u_0 and g are nonnegative.

The Volterra equation (V) is an abstraction of a mathematical model for nonlinear heat flow in a material with "memory". The kernel b in (V) can be expressed in terms of the physically meaningful heat flux and internal energy relaxation functions.

In this paper ~~we consider~~ equation (V) for a class of kernels b is considered which insure the positivity of the solution operator. For this class of kernels we use techniques of nonlinear functional analysis to establish boundedness and asymptotic behaviour of solutions of (V) as $t \to \infty$. We show that in the special case $b \in L^1(0,\infty)$ the memory induces a damping effect on solutions of (V).

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

ON ABSTRACT VOLTERRA EQUATIONS WITH
KERNELS HAVING A POSITIVE RESOLVENT

Ph. Clément

1. Introduction

Let X be a real Banach space with norm $\|\cdot\|$. Let A be a m-accretive operator in X, [3], i.e. for every $\lambda > 0$, $J_\lambda := (I + \lambda A)^{-1}$ is a non-expansive map which is everywhere defined on X. We consider the following Volterra equation of convolution type:

$$(1.1) \qquad u(t) + b * Au(t) \ni f(t) \qquad t \geq 0$$

where b is a given real kernel, f is a given function with values in X and $b * Au(t) = \int_0^t b(t-s)Au(s)ds$. Since for every $\lambda > 0$, the Yosida approximation of A, $A_\lambda := \lambda^{-1}(I-J_\lambda)$ is Lipschitz continuous, the equation

$$(1.1)_\lambda \qquad u(t) + b * A_\lambda u(t) = f(t) \qquad t \geq 0$$

possesses a unique solution $u_\lambda \in C([0,T];X)$ if $b \in L^1[0,T]$ and $f \in C([0,T];X)$, $T > 0$. In [4], Crandall and Nohel have proved that if the assumption

$$(H1) \qquad \begin{cases} b \in W^{1,1}[0,T],\ b(0) > 0,\ \dot{b} \in BV[0,T] \\ \\ f \in W^{1,1}[0,T;X],\ f(0) \in \overline{D(A)} \end{cases}$$

is satisfied, then there exists $u \in C([0,T];X)$ such that $\lim_{\lambda \downarrow 0} u_\lambda = u$ in $C([0,T];X)$; u is called the <u>generalized solution</u> of (1.1). Note that if (H1) is satisfied, then there exists a unique $u_0 \in \overline{D(A)}$ and a unique $g \in L^1(0,T:X)$ such that

$$(1.2) \qquad f(t) = u_0 + b * g(t) \qquad 0 \leq t \leq T .$$

Indeed $u_0 = f(0)$ and g is the unique solution of the equation

$$b(0)g(t) + \dot{b} * g(t) = \dot{f}(t) \qquad 0 \leq t \leq T ,$$

(where $\cdot = d/dt$). Conversely, if $b \in W^{1,1}[0,T]$, $b(0) > 0$, $\dot{b} \in BV[0,T]$ and $u_0 \in \overline{D(A)}$, $g \in L^1(0,T;X)$, then f given by (1.2) satisfies assumption (H1).

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The proof in [4] of the existence of a generalized solution of (1.1), shows that (1.1) is closely related to the equation

$$(1.3)\qquad \begin{cases} \dot{u}(t) + Au(t) \ni g(t) & 0 < t \le T \\ u(0) = u_0 \end{cases}$$

which is (1.1) with $b \equiv 1$. It is known [1], that if u_1 and u_2 are the generalized solutions of (1.3) corresponding to the data $u_{0,1}$, $u_{0,2}$ and g_1, g_2, then the following estimate, which implies continuous dependence of solutions of (1.3) holds:

$$(1.4)\qquad \|u_1(t) - u_2(t)\| \le \|u_{0,1} - u_{0,2}\| + b * \|g_1 - g_2\|(t)$$

on $[0,T]$, with $b \equiv 1$. In this paper we consider a class of kernels satisfying (H1), containing the kernel $b \equiv 1$, for which the estimate (1.4) still holds. Such class of kernels was introduced in [2, assumptions H4, H5]. Moreover, we prove that if the kernel b belongs to this class and is in $L^1(0,\infty)$, then the generalized solution of (1.1) converges strongly to a limit u_∞ provided that g itself is bounded and converges to a limit g_∞. If $b \notin L^1(0,\infty)$, it is well-known that u may not converge to a limit. (Take $X = \mathbb{R}^2$ with the Euclidean norm, $A := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $b \equiv 1$, $g = 0$, $u_0 \neq 0$). Work is in progress on an analogous result in the case $b \notin L^1(0,\infty)$ and $A = \omega I + B(\omega > 0$, B m-accretive).

In order to state our main assumption on the kernel b we need the following definitions. For $b \in L^1(0,T)$, let us denote by $r(b)$ the resolvent of b, i.e. the unique solution in $L^1(0,T)$ of the equation

$$(1.5)\qquad r + b * r = b \qquad 0 \le t \le T,$$

and by $s(b)$, the unique solution in $AC[0,T]$ of the equation

$$(1.6)\qquad s + b * s = 1 \qquad 0 \le t \le T.$$

Our basic assumption on the kernel b is

$$(\mathrm{H2}) \quad \begin{cases} \text{For every } \lambda > 0 , \ r(\lambda b) \geq 0 \ \text{a.e. on } [0,T] \\ \text{and } s(\lambda b) \geq 0 \ \text{on } [0,T]. \end{cases}$$

It is known [7], [5], [2] that if $b \in L^1(0,T)$, is positive, nonincreasing and if $\log b$ is convex on $(0,T)$, then b satisfies (H2). Observe that if b is completely monotonic on $(0,\infty)$, then $\log b$ is convex [7]. Observe also that (H2) implies $b \geq 0$. In order to avoid trivialities we shall assume that b is not identically equal to 0. In connection with this class of kernels we mention the following "positivity" result:

Theorem [2; Theorem 5] *Let* b,f *satisfy* (H1) *and* (H2) *on* $[0,T]$ *with* $f = u_0 + b * g$. *Let* P *be a closed convex cone in* X. *If* $J_\lambda(P) \subseteq P$ *for every* $\lambda > 0$, $u_0 \in P$ *and* $g(t) \in P$ a.e. *on* $[0,T]$, *then* u *the generalized solution of* (1.1) *satisfies* $u(t) \in P$, $t \in [0,T]$.

2. Statement of results

We first give the generalization of (1.4) to (1.1) with kernels b satisfying (H2).

Theorem 1. *Let* b, f_1,f_2 *satisfy* (H1) *and* (H2) *on* $[0,T]$, *with* $f_i = u_{0,i} + b * g_i$, $i = 1,2$. *Let* u_1,u_2 *be the corresponding generalized solutions of* (1.1) *on* $[0,T]$. *Then*

$$(2.1) \qquad \|u_1(t) - u_2(t)\| \leq \|u_{0,1} - u_{0,2}\| + b * \|g_1 - g_2\|(t)$$

$0 \leq t \leq T$ *holds.*

Our main result concerns the asymptotic behaviour of solutions of (1.1) as $t \to \infty$. For results in this direction, in the scalar case, but for more general kernels b, we refer the reader to [6].

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Theorem 2. *Let* b,f *satisfy* (H1) *and* (H2) *on* $[0,T]$ *for every* $T > 0$ *, with* $f = u_0 + b * g$ *and* $b \not\equiv 0$. *If* $b \in L^1(0,\infty)$, $g \in L^\infty(\mathbb{R}^+,X)$ and $\lim_{t\to\infty} g(t) = g_\infty$ *exists in* X *, then*

$$(2.2) \qquad \|u(t) - u_\infty\| \le \frac{\int_t^\infty b(s)ds}{\int_0^\infty b(s)ds} \|u_0 - u_\infty\| + b * \|g - g_\infty\|(t)$$

holds for $t > 0$ *, where* u *is the generalized solution of* (1.1) *and* $u_\infty = (I + \bar{b}A)^{-1}(u_0 + \bar{b}g_\infty)$ *with* $\bar{b} = \int_0^\infty b(s)ds$.

3. *Proofs.*

In the proofs we shall use the fact that if $v \in L^1(0,T;X)$ satisfies

$$(3.1) \qquad v(t) + b * v(t) = u_0 + b * g(t) \qquad 0 \le t \le T$$

with $b \in L^1(0,T)$, $u_0 \in X$ and $g \in L^1(0,T;X)$ then

$$(3.2) \qquad v(t) = s(b)(t)u_0 + r(b) * g(t) \qquad 0 \le t \le T$$

holds.

Proof of Theorem 1.

We first establish (2.1) with A replaced by A_λ, $\lambda > 0$ and then we pass to the limit as $\lambda \downarrow 0$. For $\lambda > 0$, let $u_{i,\lambda}$ satisfy

$$(3.3) \qquad u_{i,\lambda} + b * A_\lambda u_{i,\lambda} = u_{0,i} + b * g_i \qquad t \in [0,T], \quad i = 1,2.$$

From the definition of A_λ , we have

$$(3.4) \qquad u_{i,\lambda} + \lambda^{-1}b * u_{i,\lambda} = \lambda^{-1}b * J_\lambda u_{i,\lambda} + u_{0,i} + b * g_i , \quad i = 1,2.$$

Using (3.2) we get

$$(3.5) \qquad u_{i,\lambda} = r(\lambda^{-1}b) * J_\lambda u_{i,\lambda} + s(\lambda^{-1}b)u_{0,i} + \lambda r(\lambda^{-1}b) * g_i, \quad i = 1,2.$$

Next we estimate $\|u_{1,\lambda} - u_{2,\lambda}\|$. Since J is nonexpansive, $s(\lambda^{-1}b)$ and $r(\lambda^{-1}b)$ are nonnegative, we obtain:

$$(3.6) \qquad \|u_{1,\lambda} - u_{2,\lambda}\| \le r(\lambda^{-1}b) * \|u_{1,\lambda} - u_{2,\lambda}\| + s(\lambda^{-1}b)\,\|u_{0,1} - u_{0,2}\| + \lambda r(\lambda^{-1}b) * \|g_1 - g_2\| .$$

We take the convolution of (3.6) with $\lambda^{-1}b$ (which is nonnegative) and we add (3.6). We have

$$(3.7) \qquad \|u_{1,\lambda} - u_{2,\lambda}\| + \lambda^{-1}b * \|u_{1,\lambda} - u_{2,\lambda}\| \le$$
$$(r(\lambda^{-1}b) + \lambda^{-1}b * r(\lambda^{-1}b)) * \|u_{1,\lambda} - u_{2,\lambda}\|$$
$$+ (s(\lambda^{-1}b) + \lambda^{-1}b * s(\lambda^{-1}b))\|u_{0,1} - u_{0,2}\|$$
$$+ \lambda(r(\lambda^{-1}b) + \lambda^{-1}b * r(\lambda^{-1}b)) * \|g_1 - g_2\|.$$

From the definition of $r(\lambda^{-1}b)$ and $s(\lambda^{-1}b)$, we obtain $\|u_{1,\lambda} - u_{2,\lambda}\| \le \|u_{0,1} - u_{0,2}\| + b * \|g_1 - g_2\|$. The conclusion of Theorem 1 follows by letting λ go to 0.

Proof of Theorem 2.

As in the proof of Theorem 1, we first prove the result with A replaced by A_λ, $\lambda > 0$ and then we pass to the limit as $\lambda \downarrow 0$.

For $\lambda > 0$, let u_λ satisfy

$$(3.8) \qquad u_\lambda + b * A_\lambda u_\lambda = u_0 + b * g.$$

From the definition of A_λ and (3.2) we have:

$$(3.9) \qquad u_\lambda = r(\lambda^{-1}b) * J_\lambda u_\lambda + s(\lambda^{-1}b)u_0 + \lambda r(\lambda^{-1}b) * g.$$

Since A is m-accretive, A_λ is also m-accretive and there is a unique $u_{\lambda\infty}$ satisfying

$$(3.10) \qquad u_{\lambda\infty} + \bar{b}\, A_\lambda u_{\lambda\infty} = u_0 + \bar{b}\, g_\infty$$

where $\bar{b} = \int_0^\infty b(s)ds$.

Using again the fact that $b \in L^1(0,T)$, we can rewrite (3.10) as

$$(3.11) \qquad u_{\lambda\infty} + b * A_\lambda u_{\lambda\infty} = u_0 + b * g + b * (g_\infty - g) - \xi\, w_\lambda$$

where

$$(3.12) \qquad \xi(t) := \int_t^\infty b(s)ds$$

and

$$(3.13) \qquad w_\lambda := A_\lambda u_{\lambda\infty} - g_\infty .$$

Let η satisfy

$$(3.14) \qquad \eta + \lambda^{-1} b * \eta = \xi .$$

Then obviously $\eta\, w_\lambda$ satisfies

$$(3.15) \qquad \eta\, w_\lambda + \lambda^{-1} b * \eta\, w_\lambda = \xi\, w_\lambda .$$

Using (3.11), (3.15), (3.2) and the definition of A_λ we obtain

$$(3.16) \qquad u_{\lambda\infty} = r(\lambda^{-1}b) * J_\lambda u_{\lambda\infty} + s(\lambda^{-1}b) u_0 + \lambda r(\lambda^{-1}b) * g + \lambda r(\lambda^{-1}b) * (g_\infty - g) - \eta\, w_\lambda .$$

Subtracting (3.16) from (3.9) and using the fact that J_λ is nonexpansive, $s(\lambda^{-1}a)$, $r(\lambda^{-1}a)$ are nonnegative, we get:

$$(3.17) \qquad \|u_\lambda - u_{\lambda\infty}\| \le r(\lambda^{-1}b) * \|u_\lambda - u_{\lambda\infty}\| + \lambda r(\lambda^{-1}b) * \|g_\infty - g\| + |\eta| \|w_\lambda\| .$$

Next we take the convolution of (3.17) with $\lambda^{-1}b$ (which is nonnegative) and we add (3.17); we obtain

$$(3.18) \qquad \|u_\lambda - u_{\lambda\infty}\| \le b * \|g - g_\infty\| + (|\eta| + \lambda^{-1} b * |\eta|) \|w_\lambda\| .$$

We claim that η is nonnegative. Indeed η satisfies (3.14) with $\xi(t) = \bar{b} - \int_0^t b(s)ds$. Thus η satisfies (3.1) for every $T > 0$, with $X = \mathbb{R}$, b replaced by $\lambda^{-1}b$, u_0 replaced by $\bar{b}$ and g replaced by $-\lambda \mathbb{1}$, where $\mathbb{1}(t) \equiv 1$. From (3.2) we get

$$(3.19) \qquad \eta(t) = s(\lambda^{-1}b)(t)\bar{b} - \lambda \int_0^t r(\lambda^{-1}b)(\tau)d\tau \qquad t > 0 .$$

By using the identity

$$(3.20) \qquad s(\lambda^{-1}b)(t) + \int_0^t r(\lambda^{-1}b)(\tau)d\tau = 1 \qquad t \ge 0$$

we have

$$(3.21) \qquad \dot{\eta}(t) = -\bar{b}\, r(\lambda^{-1}b)(t) - \lambda r(\lambda^{-1}b)(t) \qquad t \ge 0 .$$

The fact that $\bar{b}, \lambda$ are positive and assumption (H2) imply that η is nonincreasing. It remains to prove that $\lim_{t\to\infty} \eta(t) \ge 0$.

From (3.20) and assumption (H2), it follows that $s(\lambda^{-1}b)(\infty) := \lim_{t\to\infty} s(\lambda^{-1}b)(t)$ exists and

$$(3.22) \qquad s(\lambda^{-1}b)(\infty) = 1 - \int_0^\infty r(\lambda^{-1}b)(\tau)d\tau$$

holds.

Letting t go to infinity in (3.19) we get

$$(3.23) \qquad \lim_{t\to\infty} \lambda^{-1}\eta(t) = s(\lambda^{-1}b)(\infty)\lambda^{-1}\bar{b} - \int_0^\infty r(\lambda^{-1}b)(\tau)d\tau$$

hence from (3.22), we obtain

$$(3.24) \qquad \lim_{t\to\infty} \lambda^{-1}\eta(t) = s(\lambda^{-1}b)(\infty)(1 + \lambda^{-1}\bar{b}) - 1 .$$

Next we observe that

$$(3.25) \qquad s(\lambda^{-1}b)(\infty)(1 + \lambda^{-1}\bar{b}) = 1 .$$

Indeed from the definition of $s(\lambda^{-1}b)$ we have

$$s(\lambda^{-1}b)(t) + \lambda^{-1}b * s(\lambda^{-1}b)(t) = 1 .$$

But $\lim_{t\to\infty} s(\lambda^{-1}b)(t)$ exists, $0 \le s(\lambda^{-1}b)(t) \le 1$ for every $t \ge 0$, and $b \in L^1(0,\infty)$, thus (3.25) follows. Consequently $\lim_{t\to\infty} \eta(t) = 0$ and $\eta(t) \ge 0$ for every $t \ge 0$. Replacing $|\eta|$ by η in (3.18) and using (3.14), we obtain:

$$(3.26) \qquad \|u_\lambda - u_{\lambda\infty}\| \le \xi\|w_\lambda\| + b * \|g - g_\infty\| .$$

Since $\bar{b} \ge 0$, using (3.13) we have

$$(3.27) \qquad \xi(t)\|w_\lambda\| = \frac{\xi(t)}{\bar{b}} \|\bar{b}\, A_\lambda u_{\lambda\infty} - \bar{b}g_\infty\| .$$

Finally using (3.10), (3.12), (3.26) and (3.27) we get

$$(3.28) \qquad \|u_\lambda(t) - u_{\lambda\infty}\| \le \frac{\int_t^\infty b(s)ds}{\int_0^\infty b(s)ds} \|u_0 - u_{\lambda\infty}\| + b * \|g - g_\infty\|(t) \qquad t \ge 0 .$$

Observe that (3.28) is the conclusion of the theorem with A replaced by A_λ.

Since A is m-accretive we have

$$\lim_{\lambda\downarrow 0} u_{\lambda\infty} := \lim_{\lambda\downarrow 0} (I + \bar{b}A_\lambda)^{-1}(u_0 + \bar{b}\, g_\infty)$$

$$= \lim_{\lambda\downarrow 0} (I + \bar{b}A)^{-1}(u_0 + \bar{b}\, g_\infty) =: u_\infty .$$

Using assumption (H1), $\lim_{\lambda \downarrow 0} u_\lambda = u$ in $C([0,T];X)$, thus (2.2) follows from (3.28) by letting λ go to 0.

Remark. It is clear from the proofs of the theorems that the assumption (H1) has been used only to insure that $\lim_{\lambda \downarrow 0} u_\lambda$ exists in $C([0,T];X)$, for every $T > 0$. Indeed Theorems 1 and 2 are valid for A replaced by A_λ, $\lambda > 0$, if the assumption (H1) is replaced by the assumption

$$(H1') \qquad \begin{cases} a \in L^1(0,T) \\ u_0 \in X, \quad g \in L^1(0,T;X) \end{cases}$$

It has been proved in [2, Th. 1(ii), Th. 2(i), Remark 2.3] that under the assumptions (H1') and (H2), if A is linear m-accretive with $D(A)$ dense in X that $\lim_{\lambda \downarrow 0} u_\lambda$ exists in $L^1(0,T;X)$. Therefore in the linear case, Theorems 1 and 2 are true with (H1) replaced by (H1'). Then pointwise inequalities (2.1) and (2.2) have to be replaced by a.e. inequalities.

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We consider the nonlinear abstract Volterra equation of convolution type:

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where A is m-accretive in a Banach space X, b is a given real kernel, $u_0 \in \overline{D(A)}$ and $g: [0,\infty) \to X$ are given.

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